

Uncertainty Shocks in a Model of Flexible Prices

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Abstract

I revisit the macroeconomic comovement puzzle studied by Basu and Bundick (2017). Increased uncertainty about the future generates contractions in output, consumption, investment, and hours worked in the data. The baseline RBC model is unable to produce the fact because labor supply curve shifts out in response to higher uncertainty but labor demand curve stays unchanged, so equilibrium hours worked increases. I propose the discount rate channel as an alternative to the markup channel in existing literature. When firms can't change labor input with perfect flexibility, hiring decisions should be based on the expected future revenues generated by the marginal labor. Future revenues are discounted more heavily when uncertainty increases, so firms reduce hiring. The reduction in labor demand offsets the increase in labor supply, so equilibrium employment decreases. Without shocks to the levels of TFP or other fundamentals, output and its components decrease.

Keywords: Uncertainty shocks, discount rate, RBC model

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1 Introduction

Many empirical studies find that the effects of an uncertainty shock are contractionary: real output, consumption, investment, and employment fall when the future is more uncertain. However, it is hard to generate such comovements in a standard RBC model. The key tension is that the precautionary motive of the household to decrease consumption leads to an increase in labor supply. Without any first moment shocks, labor demand curve stays unchanged, so equilibrium employment and output increase. In order to be consistent with the empirical facts, the model has to produce inward shift of the labor demand curve. In this paper, I show that when the labor demand is forward-looking, an increase in uncertainty has contractionary effects because it reduces the stochastic discount factor. The firm discounts more heavily the future value of the marginal labor, so labor demand shifts to the left.

Intuitively, the contractionary effects of increased uncertainty are straightforward. Decision makers are risk averse. Risky cashflows are less valuable when uncertainty is higher, so investors cut back investment in risky projects. The standard RBC model is unable to generate the contractions in response to increased uncertainty. The key tension is in the labor market. Perfectly flexible labor market makes the labor demand in the standard RBC model static, so the expected future value of the marginal labor has no effect on labor demand. Therefore, the “valuation effect” of uncertainty on labor demand is silent in the RBC model. Given the level of TFP unchanged and the capital stock predetermined, the change in equilibrium employment can only be the consequence of the change in the labor supply curve. Households would like to reduce consumption and work more when faced with higher uncertainty, so equilibrium employment increases. Output increases and investment has to increase to clear the market in the closed economy.

Labor market frictions such as training costs and search frictions make the labor market far from fully flexible. Firms would expect to employ a worker for extended period of time even when the marginal product of labor falls lower than the real wage. Hiring decisions should incorporate such expectation, so the employer evaluates both the current marginal product of labor and the expected future value of the worker, properly discounted by the stochastic discount factor. Under higher uncertainty, the market discounts risky cashflows by a larger discount rate, depressing the expected value of the marginal labor. Therefore, hiring decreases and the economy falls into a contraction.

Using a calibrated DSGE model, I show that labor adjustment cost reduces labor demand in

response to increased uncertainty. The model is an otherwise standard RBC model in which all markets are competitive and prices are flexible. The expected value of labor is a novel channel relative to the literature. Most, if not all, of the existing literature that studies the contractionary effects of second moment shocks in a representative agent model adopts a New Keynesian framework. As Basu and Bundick (2017) emphasizes, the demand-driven production in a New Keynesian model is the key to the contraction in labor demand. Since households demand less consumption, the firms produce less to match the demand. The firms' markup increase in response to the increase in labor supply, so labor demand shifts leftward. Alternatively, I show that the reduction in the expected value of the marginal labor is able to reduce labor demand in a competitive and flexible price environment.

In order to compare with the commonly adopted New Keynesian framework, I augment the Basu and Bundick (2017) model with a quadratic labor adjustment cost, and eliminate price stickiness. Other model structures and the parameter values are the same as Basu and Bundick (2017). In spite of the flexible prices, my model generates similar comovement patterns with Basu and Bundick (2017). Moreover, my model generates larger volatility in hours worked than Basu and Bundick (2017). Labor demand in my model is sensitive to the expected future value of the marginal labor, which is volatile because of the recursive preference and the uncertainty shock. It's been noted that many standard macroeconomic models struggle to generate sufficient fluctuations in hours worked. The fluctuations in the expected future value of labor provide a possible solution to this puzzle.

Related literature Basu and Bundick (2017) is the closest to this paper. They argue that the market clearing conditions in a flexible price model make the effects of uncertainty shocks expansionary, and that the endogenous increase in markups in a sticky price model generates the correct responses of macroeconomic aggregates to uncertainty shocks. Relying on the same mechanism, Fernández-Villaverde et al. (2015) and Born and Pfeifer (2014) show that higher uncertainty in fiscal policy has contractionary effects on economic activities. Importantly, labor demand decisions in these papers are static. Leduc and Liu (2016) show that TFP volatility shocks could have contractionary effects in a labor search model. Higher uncertainty reduces hiring by increasing the option value of waiting. Similar mechanism arises in the literature on irreversible investment / hiring decisions under uncertainty (Bloom et al. (2018), Bloom (2009)). Labor decisions in these papers are forward-looking, but the stochastic discount factor plays secondary roles. Hall (2017)

relates the asset price fluctuations with labor market dynamics through the stochastic discount factor. However, he does not focus on the effects of the uncertainty shocks on macroeconomic comovements.

2 Stylized empirical fact

Figure 2.1 shows a stylized fact: an increase in uncertainty about the future has contractionary effects. The figure shows impulse responses to a positive one-standard-deviation shock to the VXO index, which reflects investors' expectations for short-term (30-day) volatility in the stock market. The impulse responses are produced by a VAR, and the structural shocks are identified by Cholesky decomposition. Following Basu and Bundick (2017) and Bloom (2009), the uncertainty measure is ordered first. The list and ordering of all variables are VXO, $\ln(\text{real GDP})$, $\ln(\text{real consumption})$, $\ln(\text{real investment})$, $\ln(\text{hours worked})$, $\ln(\text{real wage})$, and real interest rate. Except for VXO and real interest rate, impulse responses of other macro variables denote percentage deviations from the steady state. Impulse responses of VXO and real interest rate denote deviations in levels. The sample period is 1986q1 - 2017q4.

The uncertainty shock increases VXO by 15 annualized percentage points. On impact, real output, consumption, investment, hours worked, real wage, and real interest rate all decrease, peaking after about 1 year. The falls in the real interest rate and consumption suggest that the households want to reduce consumption and increase saving. On the firm side, since there is no change in the level of productivity or predetermined capital stock, the movements in the household sector seem to be favorable to expansions in output and investment. The negative impulse responses of real output and investment suggest that there are factors reducing the demand for labor and investment in the firm sector.

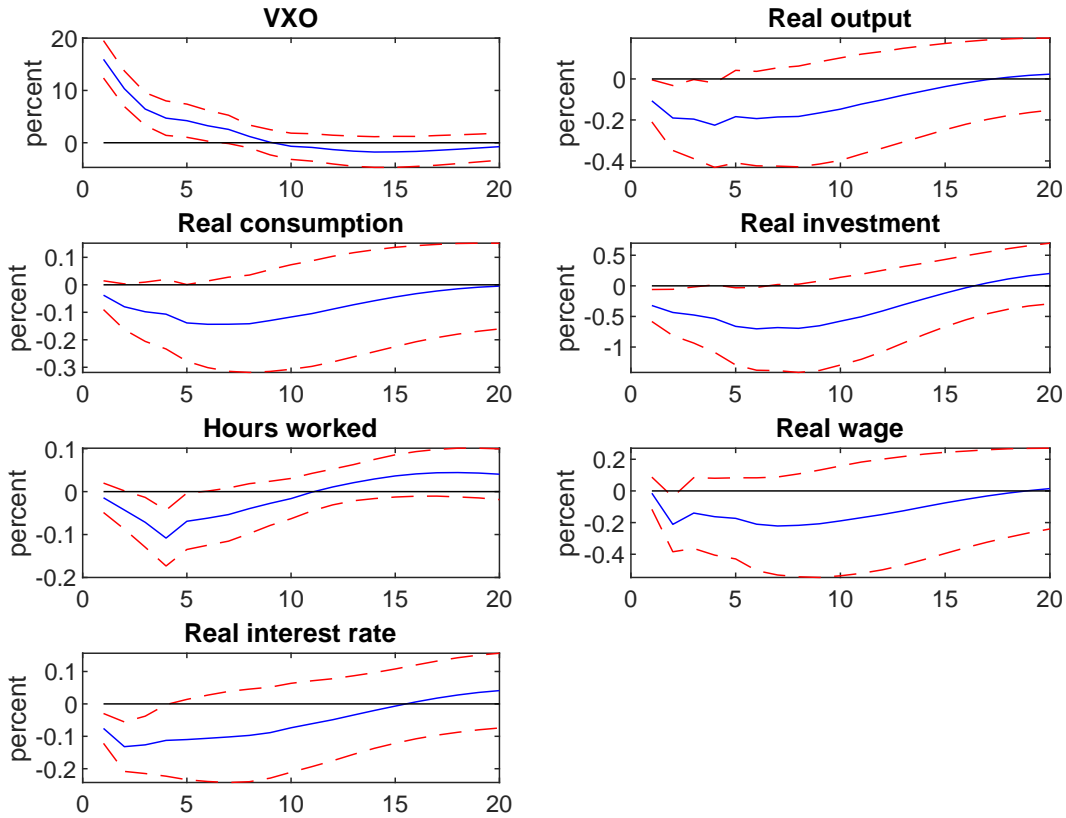


Figure 2.1: Impulse responses to an increase in uncertainty. Dashed lines denote 90% confidence intervals. VXO and real interest rate are measured as deviations from steady state in levels. Other variables are measured as percentage deviations.

3 Baseline model

The model is a standard RBC model augmented by a labor adjustment cost. The role of labor adjustment cost is to make the labor demand a dynamic decision. When the firm hires an additional labor, it considers not only the current marginal product of the labor, but also the marginal effects on future paths of the cost of adjustment. The paths of future adjustment costs should be properly discounted by the stochastic discount factor, which is highly sensitive to uncertainty shocks. In this way, aggregate labor demand does not only depend on the level of TFP, but also the expected

marginal value of the marginal labor. This mechanism is in contrast to Basu and Bundick (2017), who rely on countercyclical movements in the markup to generate demand recessions responding to an increase in uncertainty. In order to show that my mechanism does not rely on the movements in markup, I assume that prices are perfectly flexible in the baseline model.

3.1 Household

The representative household has Epstein-Zin preference over streams of consumption $\{C_t\}_{t \geq 0}$ and leisure $\{1 - N_t\}_{t \geq 0}$. The risk aversion parameter is γ and the elasticity of intertemporal substitution is ψ . When $\gamma > (<) 1/\psi$, the household prefers early (late) resolution of uncertainty. Note that when $\gamma = 1/\psi$, the preference is equivalent to a time-separable CRRA preference. Preference over consumption also features external habit persistence, given by χC_{t-1} where C_{t-1} denotes aggregate consumption of the previous period. The household takes C_{t-1} as given and doesn't take into account the effect of its own consumption on next period's habit.¹ The representative household trades shares of the firm's stock S_t and risk-free bonds B_t in the financial market. Stocks have a price of P_t^E and pay dividends D_t^E for each share. The bond pays one unit of consumption good at maturity and the gross interest rate is R_t^R . The total number of stocks is normalized to 1 and the bond has zero net supply. The representative household maximizes recursive utility by choosing $C_t, N_t, B_{t+1}, S_{t+1}$:

$$V_t = \max \left[a_t \left((C_t - \chi C_{t-1})^\eta (1 - N_t)^{1-\eta} \right)^{1-\frac{1}{\psi}} + \beta \left(\mathbf{E}_t \left[V_{t+1}^{1-\gamma} \right] \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\psi}}} \quad (3.1)$$

subject to the intertemporal household budget constraint each period

$$C_t + P_t^E S_{t+1} + \frac{1}{R_t^R} B_{t+1} = W_t N_t + (D_t^E + P_t^E) S_t + B_t. \quad (3.2)$$

¹The main reason for introducing habit persistence is to generate the hump-shaped impulse response of consumption. It is not crucial for the comovements among the aggregate variables.

It can be shown that the stochastic discount factor $M_{t,t+1}$ between periods t and $t + 1$ is

$$M_{t,t+1} = \left(\beta \frac{a_{t+1}}{a_t} \right) \left(\frac{(C_{t+1} - \chi C_t)^\eta (1 - N_{t+1})^{1-\eta}}{(C_t - \chi C_{t-1})^\eta (1 - N_t)^{1-\eta}} \right)^{1 - \frac{1}{\psi}} \frac{C_t - \chi C_{t-1}}{C_{t+1} - \chi C_t} \left(\frac{V_{t+1}^{1-\gamma}}{\mathbf{E}_t [V_{t+1}^{1-\gamma}]} \right)^{1 - \frac{1}{1-\gamma}}. \quad (3.3)$$

The stochastic discount factor is used to discount risky cashflows. Since it is multiplied with the cashflows, it is inversely related with the “discount rate” that is commonly thought to divide the risky cashflows. Indeed, no-arbitrage condition implies that the return on any asset R^a should satisfy the equation $1 = \mathbf{E}_t [M_{t,t+1} R_{t+1}^a]$. In the rest of the paper, “a decrease in the stochastic discount factor” is used interchangeably with “a rise in the discount rate”. The intratemporal first-order condition for labor supply is

$$(1 - \eta)(C_t - \chi C_{t-1}) = \eta W_t (1 - N_t). \quad (3.4)$$

3.2 Firm

The representative firm owns capital, and hires labor in the labor market. Capital accumulation and labor hiring are subject to a joint convex adjustment cost function $\Phi(I_t, K_t, H_t, N_{t-1})$:

$$\Phi = \frac{\phi_1}{2} \left(\frac{I_t}{K_t} - \delta_{K,0} \right)^2 K_t + \frac{\phi_2}{2} \left(\frac{H_t}{N_{t-1}} - \delta_N \right)^2 N_{t-1}$$

where I_t denotes investment, H_t denotes hiring. The production function has constant returns to scale:

$$Y_t = (U_t K_t)^\alpha (Z_t N_t)^{1-\alpha}$$

where Z_t is the TFP, and U_t is capital utilization rate chosen by the firm. Assume that the firm is an all-equity firm, so the dividend payment is

$$D_t^E = Y_t - W_t N_t - I_t - \Phi(I_t, K_t, H_t, N_{t-1}).$$

The firm optimizes the discounted cashflows by choosing $\{I_t, H_t\}_{t \geq 0}$

$$J_t = \max \mathbf{E}_t \sum_{s=1}^{\infty} M_{t,t+s} D_{t+s}^E \quad (3.5)$$

subject to the capital and labor accumulation equations

$$K_{t+1} = I_t + (1 - \delta_K(U_t)) K_t, \quad (3.6)$$

$$N_t = H_t + (1 - \delta_N) N_{t-1}. \quad (3.7)$$

Note that new capital created by investment becomes available in the next period, but new labor hired is available right away. Since one period in the model represents one quarter, it is reasonable to assume that new labor becomes available in the current quarter. Finally, capital depreciation rate $\delta_K(U_t)$ depends on the current capital utilization rate. The functional form of $\delta_K(U_t)$ is

$$\delta_K(U_t) = \delta_{K,0} + \delta_{K,1}(U_t - U) + \frac{\delta_{K,2}}{2}(U_t - U)^2.$$

First order conditions The Bellman equation for the firm's problem is

$$\begin{aligned} J(K_t, N_{t-1}) &= \max_{I_t, H_t, U_t, K_{t+1}, N_t} [Z_t (U_t K_t)^\alpha N_t^{1-\alpha} - W_t N_t - I_t - \Phi(I_t, K_t, H_t, N_{t-1})] + \mathbf{E}_t [M_{t,t+1} J(K_{t+1}, N_t)] \\ \text{s.t. } K_{t+1} &= I_t + (1 - \delta_K(U_t)) K_t \\ N_t &= H_t + (1 - \delta_N) N_{t-1}. \end{aligned}$$

Let the Lagrange multipliers for the capital and labor evolution equations be λ_t^K and λ_t^N . Then

$$-1 - \Phi_I + \lambda_t^K = 0 \quad (3.8)$$

$$-\Phi_H + \lambda_t^N = 0 \quad (3.9)$$

$$\alpha Z_t (U_t K_t)^{\alpha-1} N_t^{1-\alpha} = [\delta_{K,1} + \delta_{K,2}(U_t - U)] \lambda_t^K \quad (3.10)$$

$$\mathbf{E}_t [M_{t,t+1} J_K(K_{t+1}, N_t)] - \lambda_t^K = 0$$

$$\mathbf{E}_t [M_{t,t+1} J_N(K_{t+1}, N_t)] + [(1 - \alpha) Z_t (U_t K_t)^\alpha N_t^{-\alpha} - W_t] - \lambda_t^N = 0.$$

Envelope condition for K_t :

$$J_K(K_t, N_{t-1}) = \alpha Z_t U_t^\alpha K_t^{\alpha-1} N_t^{1-\alpha} - \Phi_K + (1 - \delta_K(U_t)) \lambda_t^K.$$

Envelope condition for N_{t-1} :

$$J_N(K_t, N_{t-1}) = -\Phi_N + (1 - \delta_N) \lambda_t^N.$$

Combining the envelope conditions with other first-order conditions, we get the Euler equations for capital and labor demand:

$$\lambda_t^K = \mathbf{E}_t [M_{t,t+1} (\alpha Z_{t+1} U_{t+1}^\alpha K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} - \Phi_K + (1 - \delta_K(U_{t+1})) \lambda_{t+1}^K)] \quad (3.11)$$

$$\lambda_t^N = (1 - \alpha) Z_t (U_t K_t)^\alpha N_t^{-\alpha} - W_t + \mathbf{E}_t [M_{t,t+1} (-\Phi_N + (1 - \delta_N) \lambda_{t+1}^N)]. \quad (3.12)$$

The latter equation can be rewritten as

$$W_t = (1 - \alpha) Z_t (U_t K_t)^\alpha N_t^{-\alpha} - \lambda_t^N + \mathbf{E}_t [M_{t,t+1} (-\Phi_N + (1 - \delta_N) \lambda_{t+1}^N)]. \quad (3.13)$$

The first term is the standard marginal product of labor, which is the condition for the static labor demand when there is no labor adjustment cost. The second and third terms in the equation reflects the fact that the current labor demand affects future adjustment costs, so the current labor demand has a dynamic consideration. When the shocks affect the SDF $M_{t,t+1}$, the firms value future cash-flows differently, so the dynamic labor demand changes. This is the key mechanism of this paper. Consider a positive uncertainty shock that increases the volatility of the TFP. The SDF decreases because the investors in the financial market have lower valuations for risky income streams, so the future income stream generated by a marginal worker is discounted more heavily. The last term in Equation 3.13 decreases, so the marginal product of labor in the current period has to increase to match any given level of wage. Graphically, the labor demand curve shifts leftwards. Hall (2017) illustrates that the same mechanism leads to high unemployment in a financial crisis.

3.3 Exogenous shocks

Following Basu and Bundick (2017), the preference shock and TFP shock processes are specified as the following:

$$\begin{aligned} Z_t &= (1 - \rho_Z)Z + \rho_Z Z_{t-1} + \sigma^Z \varepsilon_t^Z, \\ a_t &= (1 - \rho_a)a + \rho_a a_{t-1} + \sigma_{t-1}^a \varepsilon_t^a, \\ \sigma_t^a &= (1 - \rho_{\sigma^a})\sigma^a + \rho_{\sigma^a} \sigma_{t-1}^a + \sigma^{\sigma^a} \varepsilon_t^{\sigma^a}. \end{aligned}$$

ε_t^Z and ε_t^a are first-moment shocks that change the levels of the preference parameter and the TFP. $\varepsilon_t^{\sigma^a}$ is the second-moment or “uncertainty” shock to the preference parameter. An increase in $\varepsilon_t^{\sigma^a}$ increases the uncertainty of the future path of household’s stochastic discount factor and the demand for consumption and leisure.

3.4 Solution and calibration methods

The model is solved by a third order approximation. This is the lowest order approximation necessary for a second moment shock to have time-varying effects. The approximation is implemented in Dynare 4.5.5. The reference point for the approximation is the deterministic steady state, while the impulse responses are characterized as percentage deviations² from the stochastic steady state. To compute the stochastic

The model parameters that have counterparts in Basu and Bundick (2017) are set equal to their counterparts to facilitate comparison. The new parameters are the parameters of the adjustment cost $\phi_1, \phi_2, \phi_3, \phi_4$ and the labor separation rate δ_N . I set $\delta_N = 0.1$ to match the steady state quarterly rate. The adjustment cost parameters are calibrated to minimize the distance between the model generated IRF and the data implied IRF:

$$\min [\hat{\Psi} - \Psi]' V^{-1} [\hat{\Psi} - \Psi].$$

$\hat{\Psi}$ denotes the empirical impulse responses in Figure 2.1, and Ψ denotes the model generated counterparts. V is a diagonal matrix with the variances of the empirical impulse responses on the main diagonal.

²Except for the real interest rate, which is characterized as deviation in levels.

Table 1: Model parameters

Household		Firm		Shocks	
β	0.994	α	0.333	ρ_a	0.94
γ	80	δ_0	0.025	σ^a	0.003
ψ	0.95	δ_1	0.03	ρ_{σ^a}	0.74
η	0.17	δ_2	0.0003	σ^{σ^a}	0.003
χ	0.6	ϕ_1	2.09	ρ_Z	0.99
		ϕ_2	5	σ^Z	0.001

4 Uncertainty shocks and the role of expectations

4.1 Effects of the uncertainty shock in the baseline model

Figure 4.1 plots the impulse response functions generated by the baseline against the empirical impulse responses. A positive uncertainty shock hits the economy. Consumption falls because of the standard precautionary motive. Households would like to increase labor supply because consumption and leisure are complements, but the equilibrium hours worked also depends on the location of the labor demand curve. When the labor adjustment cost is present, the value of the marginal labor to the firm consists of the current marginal product of labor plus the change in the expected paths of adjustment costs in the future. An increase in the uncertainty about future decreases the expected future value of the marginal labor, so labor demand curve shifts to the left, decreasing equilibrium hours worked. With no change in the TFP level and predetermined capital stock, output falls. Investment also falls because the expected value of future capital falls.

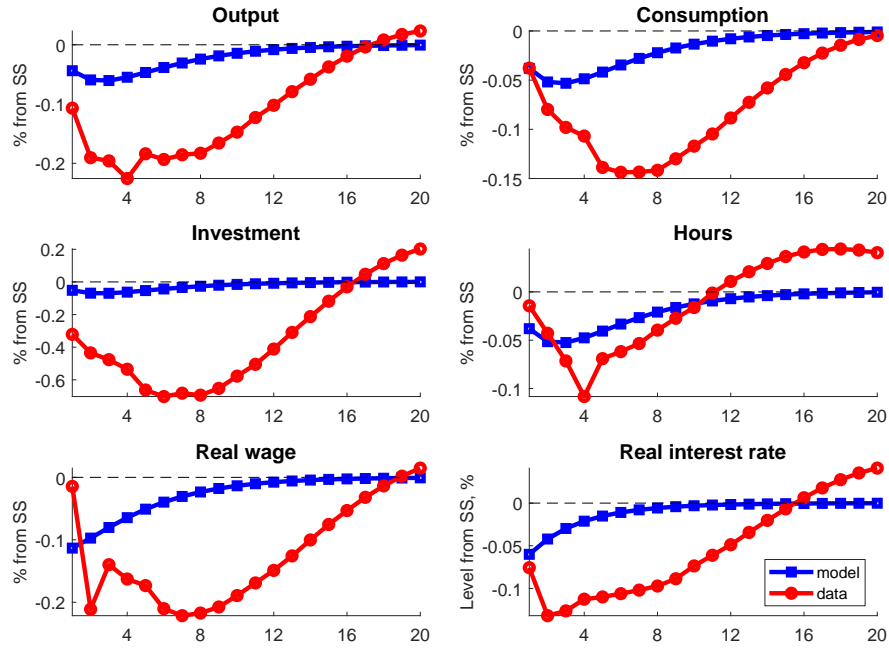


Figure 4.1: Impulse responses to one-standard-deviation increase in preference shock volatility, baseline model. Interest rate is expressed as deviations from the steady state in levels, annualized percentage points. Other variables are expressed as percentage deviations from the steady state, not annualized.

4.2 No adjustment cost for labor

Figure 4.2 plots the impulse response functions to an increase in uncertainty in a model without labor adjustment cost and the interacted adjustment costs. That is, $\phi_2 = 0$ with other parameters unchanged. Consumption behaves similarly to the baseline model due to the precautionary motive. Having no effects on the adjustment costs, labor demand decision is static. This case is the same as the flexible price model studied by Basu and Bundick (2017). Labor demand curve staying unchanged, the precautionary motive of the household increases labor supply and increases hours worked. Capital stock and TFP level are unchanged, so output increases. In the closed economy equilibrium, consumption and investment sum up to output, so investment has to increase.

The contrast with the baseline model shows that the expected value of the marginal labor may have significant effects on labor demand and other macroeconomic variables. When labor demand is static, the desire to decrease consumption leads to an expansion in output and investment through the increase in hours worked. When labor demand involves dynamic considerations, an

increase in uncertainty can have contractionary effects even if prices are flexible.

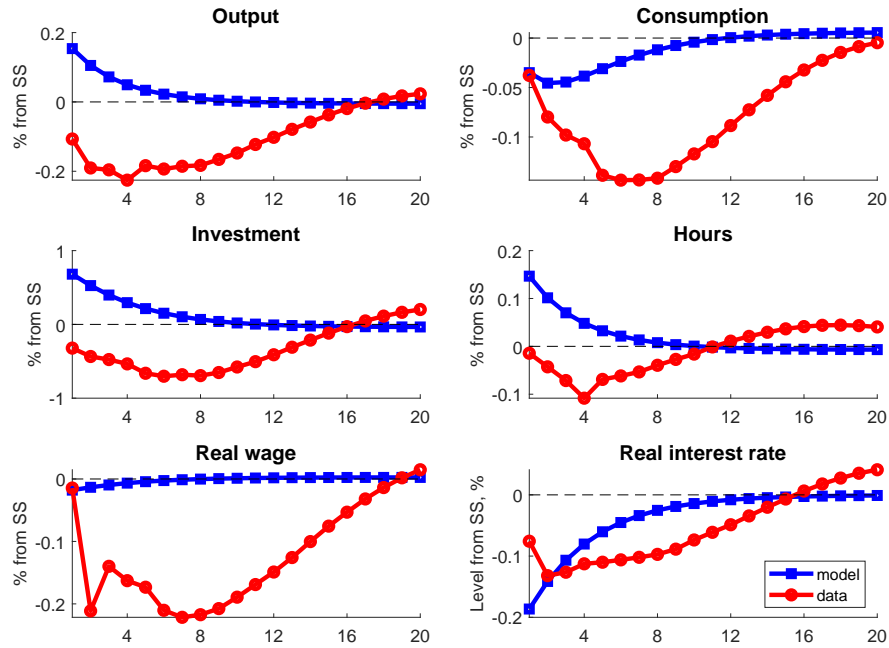


Figure 4.2: Impulse responses to one-standard-deviation increase in preference shock volatility. Adjustment cost involves capital adjustment cost only. Interest rate is expressed as deviations from the steady state in levels, annualized percentage points. Other variables are expressed as percentage deviations from the steady state, not annualized.

4.3 Risk-neutral firms

In the model, both the representative household and the firm are risk averse, in the sense that they both discount the future by the stochastic discount factor. The sharp decline in the stochastic discount factor leads to low valuations of investment and employment, so the economic aggregates contract. Alternatively, other mechanisms could also contribute to the contractionary effects of the uncertainty shock. One mechanism is the standard argument that the contraction in household demand for output leads to less production. Another mechanism is that the uncertainty shock increases the spread of future income streams, so the expected future value of the marginal labor could decrease. Similar mechanism has been emphasized by Leduc and Liu (2016).

In order to inspect the alternative mechanisms, we need to shut down the stochastic discount factor channel on the firm side while keeping other parts of the model intact. Most crucially,

the household's demand for consumption and labor supply should not change. Therefore, it is inappropriate to simply replace the household's preference by one less sensitive to uncertainty, such as the risk-neutral preference. Doing so, household's Euler equation and labor supply decision would change sharply.

To isolate the effect of the stochastic discount factor on the firm sector, I assume that the representative firm discounts dividend streams by the real interest rate and all other parts of the model are the same as the baseline model. By such construction the firm resembles a risk neutral investor who discounts risky cashflows by the real interest rate. This is a counterfactual exercise because the value of the firm's equity, discounted by the household's SDF, is inconsistent with the way the firm's manager values the future cashflows so the firm manager is not maximizing expected value of the firm from the owner's perspective. This counterfactual is constructed so that the future value of the marginal labor still matters for labor demand, but the discount rate is different from the SDF in the financial market. If the recession in household demand for output dominates, then it shouldn't matter whether the firm discounts the dividend streams by the SDF or the real interest rate, and we should see contractionary effects of the uncertainty shock that are qualitatively similar with those in the baseline model. On the other hand, if the increase in the spread of the dividend streams reduces the expected value, it shouldn't matter whether the dividend streams are discounted by the stochastic discount factor or the real interest rate.

To investigate the alternative mechanisms, I modify the firm's objective function 3.5 as

$$J_t = \max \mathbf{E}_t \left[\sum_{s=1}^{\infty} \frac{1}{R_{t+s}} D_{t+s}^E \right].$$

Compared with the baseline model, the firm becomes "risk neutral" and all other parts of the model are the same. The impulse responses are shown in Figure 4.3. The numerical exercise rejects both hypotheses. The uncertainty shock has expansionary effects on output and its components when the firm is risk neutral. Consumption falls due to the precautionary motive. The desire to increase saving pushes the real interest rate down, decreasing the discount rate used by the employer. It seems that the decline in the real interest rate dominates the Jensen's inequality effect of an increase in the spread of the dividend streams. Consequently, the marginal labor has larger expected value to the firm and equilibrium hours worked increases. The uncertainty shock produces expansionary effects on output, investment, and hours worked because the firms are no longer precautionary facing increased uncertainty. The decrease in the real interest rate induces the firm to hire and

invest more aggressively.

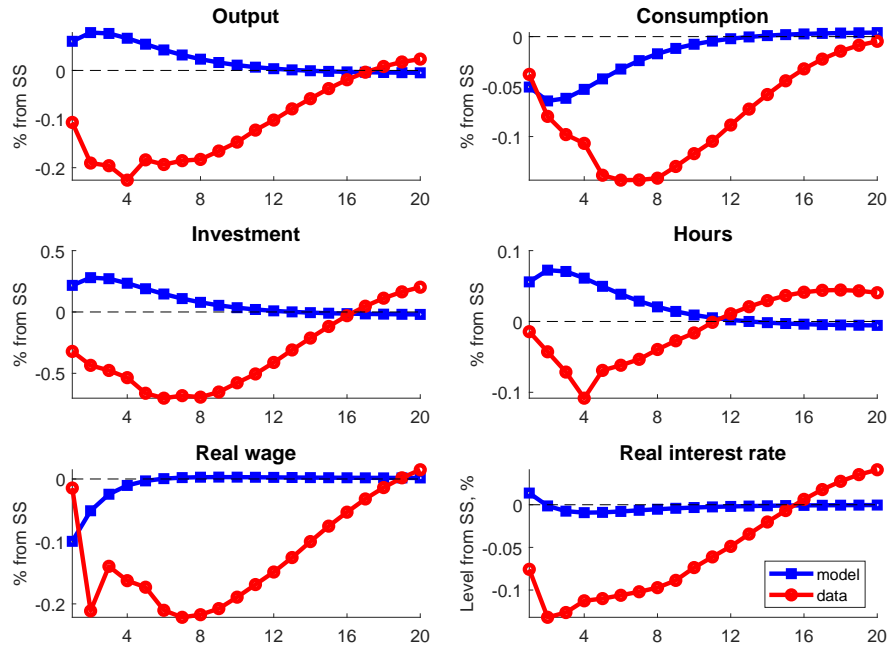


Figure 4.3: Impulse responses to one-standard-deviation increase in preference shock volatility. The firms discount future dividend payments by the real interest rate. Interest rate is expressed as deviations from the steady state in levels, annualized percentage points. Other variables are expressed as percentage deviations from the steady state, not annualized.

5 Adding price rigidity

Most of the literature on the business cycle effects of uncertainty shock relies on the endogenous increase in markups to generate contractionary effects (Basu and Bundick (2017), Fernández-Villaverde et al. (2015), Born and Pfeifer (2014)). In order to generate endogenous variations in markups, price or wage stickiness is necessary. In contrast, the baseline model of this paper does not rely on the endogenous increase in markups to generate contractionary effects of the uncertainty shock. It is interesting to compare the “markup channel” with the “discount factor” channel in terms of their abilities to generate macroeconomic comovements.

Details of the model are relegated to the appendix. Here, I briefly describe the model in words. The household block is the same as in Section 3, where the relative prices and quantities in Equations 3.1 and 3.2 are all real variables. Importantly, the risk-free bond pays back one unit of real

consumption good on maturity. There is also a nominal bond in zero net supply, whose gross return R_t^n is controlled by the monetary authority.

The producers consist of a representative final goods producer and a continuum of monopolistic competitive intermediate goods producers. The intermediate goods producers own capital and hire labor from the representative household to produce with the technology specified in Subsection 3.2. The prices of the intermediate goods are set in terms of the nominal unit of account, subject to a quadratic cost of changing prices.

The monetary authority sets the nominal interest rate according to the rule

$$r_t^n = r^n + \rho_\pi (\pi_t - \pi) + \rho_y \Delta y_t,$$

where $r_t^n = \ln(R_t^n)$ denotes the net nominal interest rate, $\pi_t = \ln(\Pi_t) = \ln(P_t/P_{t-1})$ denotes the net inflation rate, and $\Delta y_t = \ln(Y_t/Y_{t-1})$ denotes the net growth rate of real output. The nominal interest rate affects expected inflation and consumption decisions through the Euler equation

$$1 = R_t^n \mathbf{E}_t \left[M_{t+1} \frac{1}{\Pi_{t+1}} \right].$$

Figure 5.1 plots the impulse responses to a positive uncertainty shock generated by Basu and Bundick (2017)³ and the baseline model described in Section 3. Basu and Bundick (2017) model is the same as the baseline model except 1) the producers are monopolistic competitive and price setting is subject to a quadratic adjustment cost, 2) there is no labor adjustment cost.

The black dashed line represents the Basu and Bundick (2017) model without sticky price, which is the same with the baseline model without labor adjustment cost presented in Section 4.2. As explained previously, the model fails to generate contractionary effects of the uncertainty shock. This is the benchmark on which Basu and Bundick (2017) and this paper try to improve.

The green line represents the sticky price model of Basu and Bundick (2017), and the blue line represents the baseline flexible price model described in Section 3. As explained in introduction section, the key tension in the standard RBC model (black dashed line) is the increase in equilibrium hours worked. The green line shows that with price stickiness, equilibrium hours decreases in response to an increase in uncertainty. Moreover, the blue line shows that the model without price stickiness but with labor adjustment cost is able to generate even larger contraction in equilibrium

³The Basu and Bundick (2017) model is modified to have habit persistence in consumption, as Equation 3.1 shows.

hours. It follows naturally that in a model with both price stickiness and labor adjustment cost, equilibrium hours falls by the most. This conjecture is confirmed by the red line.

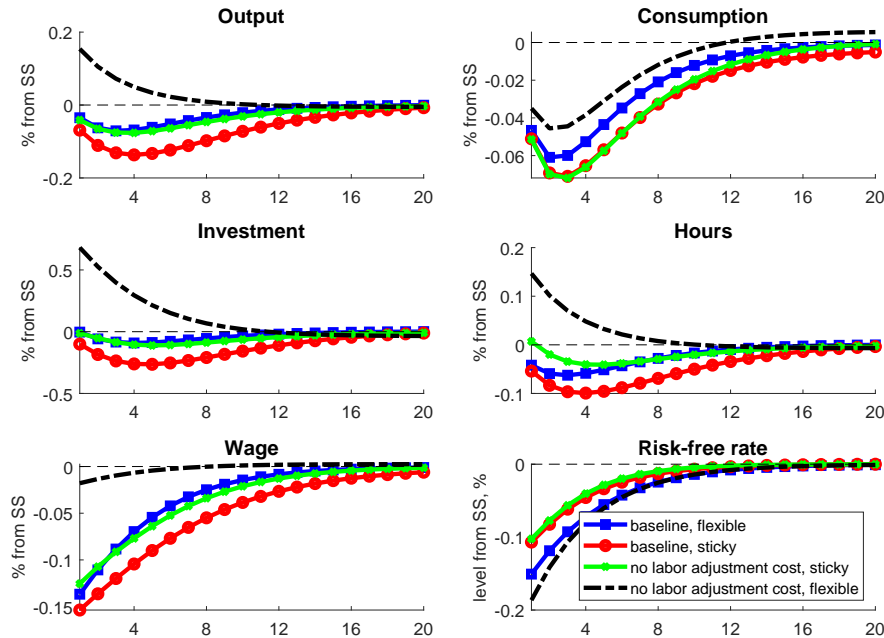


Figure 5.1: Comparing flexible and sticky price models. The “baseline, flexible” model is the model described in Section 3, the “baseline, sticky” model adds price stickiness to that model. The “no labor adjustment cost, sticky” model replicates Basu and Bundick (2017), and the “no labor adjustment cost, flexible” model is the flexible price version of that model. Interest rate is expressed as deviations from the steady state in levels, annualized percentage points. Other variables are expressed as percentage deviations from the steady state, not annualized.

6 Comparing moments

Table 2 presents the volatility of macroeconomic variables generated by different models. “NC” represents “NeoClassical” models, featuring flexible prices; “NK” represents “New Keynesian” models, featuring sticky prices. “M2, NC” is the standard RBC model presented in Subsection 4.2. “M2, NK” is the standard RBC model augmented by sticky prices, which is the baseline model studied by Basu and Bundick (2017). “M1, NC” is the baseline model introduced in Section 3, featuring forward-looking labor demand. “M1, NK” adds sticky prices to the model. Finally, “M3, NC” and “M3, NK” are the models with risk neutral firms studied in Subsection 4.3. In general, my

baseline model is able to match the unconditional volatility of major macroeconomic aggregates. The baseline flexible price model is also able to match the stochastic volatility better than the New Keynesian model of Basu and Bundick (2017).

As noted by Basu and Bundick (2017), price stickiness amplifies the impulse responses of consumption. The amplification is due to countercyclical movements in the markup. Therefore, the sticky price models generate higher volatility of consumption than the flexible price versions. Sticky price is unable to, however, generate higher volatility of hours worked. In fact, it is lower in the sticky price models. The baseline model with labor adjustment cost generates larger volatility in hours worked (1.03 percent) than the RBC model without labor adjustment cost does (0.97 percent), while simply adding price stickiness to the RBC model reduces the unconditional volatility of hours worked to 0.81 percent. The baseline model is closer to the target in the data (1.4 percent) than the model without forward looking labor demand, with or without price stickiness. Intuitively, the adjustment cost should make employment adjustment more sluggish so the volatility should be smaller. The increased volatility is due to the expectation component in labor demand. The stochastic discount factor produced by Epstein-Zin preference contains conditional expectation about future utility, which is very sensitive to shocks. The sharp reactions of the discount rate to shocks makes the expected value of future labor, which is discounted by the stochastic discount factor, very sensitive to shocks.

Table 2: Empirical and model-implied volatility of macroeconomic aggregates.

Moment	Percent					
	Data	M1, NC	M1, NK	M2, NC	M2, NK	M3, NK
Unconditional Volatility						
Output	1.1	1.34	1.22	1.19	1.01	1.35
Consumption	0.7	0.76	0.78	0.68	0.71	0.77
Investment	3.8	3.82	3.48	5.41	4.70	4.01
Hours Worked	1.4	1.03	0.94	0.97	0.81	1.04
Stochastic Volatility						
Output	0.4	0.31	0.28	0.27	0.24	0.32
Consumption	0.2	0.18	0.19	0.15	0.17	0.18
Investment	1.6	0.96	0.87	1.34	1.21	1.00
Hours Worked	0.5	0.27	0.24	0.25	0.21	0.27
Relative to Output						
Unconditional Volatility						
Output	1	1	1	1	1	1
Consumption	0.6	0.57	0.64	0.57	0.70	0.57
Investment	3.4	2.86	2.86	4.54	4.63	2.96
Hours Worked	1.3	0.77	0.77	0.81	0.80	0.77
Stochastic Volatility						
Output	1	1	1	1	1	1
Consumption	0.5	0.59	0.68	0.57	0.70	0.59
Investment	4	3.08	3.09	4.98	5.14	3.17
Hours Worked	1	0.86	0.86	0.91	0.89	0.85

NC: Neoclassical model with flexible price and adjustment costs in capital and labor.

NK: New Keynesian model with sticky price and adjustment costs in capital and labor.

M1: Baseline model. M2: No labor adjustment cost. M3: Firm discounts with the real interest rate.

Stochastic volatility is the standard deviation of a time series estimate of the 5-year rolling standard deviation.

7 Conclusion

An increase in uncertainty can lead to recessions in components of GDP if labor demand is forward-looking and risk averse. When the uncertainty increases, the stochastic discount factor decreases so the firm discounts more heavily the future value of a marginal labor, decreasing labor demand and equilibrium employment. In contrast to existing literature, time-varying markups are not necessary to generate the observed macroeconomic comovements. An interesting direction for future research is to relate the endogenous variations in the stochastic discount factor to financial crises, and explain why employment and output fell so much relative to any plausible drop in the level of TFP. As documented by Muir (2017), Hall (2017), and He and Krishnamurthy (2013), the sharp rise in the discount rate seems to be a good answer and it's interesting to incorporate it in a DSGE model.

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A The model with sticky prices

A.1 Household

The representative household solves the following problem

$$V_t = \max \left[a_t \left((C_t - \chi C_{t-1})^\eta (1 - N_t)^{1-\eta} \right)^{1-\frac{1}{\psi}} + \beta \left(\mathbf{E}_t \left[V_{t+1}^{1-\gamma} \right] \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\psi}}} \quad (\text{A.1})$$

subject to the intertemporal household budget constraint each period

$$C_t + P_t^E S_{t+1} + \frac{1}{R_t^R} B_{t+1} + \frac{1}{R_t^n} B_{t+1}^n = W_t N_t + (D_t^E + P_t^E) S_t + B_t + \frac{1}{\Pi_t} B_t^n. \quad (\text{A.2})$$

It should be understood that all the terms are expressed in terms of the real consumption good, in order to keep the notations the same as in Section 3. Alternatively, the variables can be expressed as a nominal variable divided by the price level. For example, $W_t = W_t^n / P_t$. As in Section 3, C denotes real consumption, N denotes hours worked, B denotes real bonds issued by the intermediate goods producers, B^n denotes nominal bonds which have zero net supply, and S denotes shares of equity of the intermediate goods producers. The real bond promises to pay one unit of consumption good on maturity, and the nominal bond promises to pay $1/\Pi_t$ units of consumption good on maturity in period t . P^E and D^E are the real price and dividend of the equity, R^R and R^n are the gross returns on the real and nominal bonds. $\Pi_t = P_t/P_{t-1}$ is the gross inflation rate, P_t is the consumer price index. The (real) stochastic discount factor M_{t+1} between periods t and $t+1$ is

$$M_{t+1} = \left(\beta \frac{a_{t+1}}{a_t} \right) \left(\frac{(C_{t+1} - \chi C_t)^\eta (1 - N_{t+1})^{1-\eta}}{(C_t - \chi C_{t-1})^\eta (1 - N_t)^{1-\eta}} \right)^{1-\frac{1}{\psi}} \frac{C_t - \chi C_{t-1}}{C_{t+1} - \chi C_t} \left(\frac{V_{t+1}^{1-\gamma}}{\mathbf{E}_t \left[V_{t+1}^{1-\gamma} \right]} \right)^{1-\frac{1}{\psi}}. \quad (\text{A.3})$$

The Euler equation for the real and nominal bonds are

$$1 = R_t \mathbf{E}_t [M_{t+1}] \quad (\text{A.4})$$

and

$$1 = R_t^n \mathbf{E}_t \left[M_{t+1} \frac{1}{\Pi_{t+1}} \right]. \quad (\text{A.5})$$

The Euler equation for the stock price is

$$P_t^E = \mathbf{E}_t \left[M_{t+1} (P_{t+1}^E + D_{t+1}^E) \right]. \quad (\text{A.6})$$

The intratemporal first-order condition for labor supply is

$$(1 - \eta)(C_t - \chi C_{t-1}) = \eta W_t (1 - N_t). \quad (\text{A.7})$$

A.2 Final goods producer

There is a representative final goods producer, who purchases intermediate goods $Y_t(i)$ from each intermediate goods producer $i \in [0, 1]$ at price $P_t(i)$, and sell the final goods at nominal price P_t . The market for final goods is perfectly competitive, but each intermediate goods producer i has monopoly power in its own market. The final goods production functions is

$$Y_t \leq \left(\int_0^1 Y_t(i)^{\frac{\theta_\mu - 1}{\theta_\mu}} di \right)^{\frac{\theta_\mu}{\theta_\mu - 1}}.$$

Given P_t and $P_t(i)$, the final goods producer solves the problem

$$\max_{Y_t, \{Y_t(i)\}_{i \in [0, 1]}} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \text{ s.t. } Y_t \leq \left(\int_0^1 Y_t(i)^{\frac{\theta_\mu - 1}{\theta_\mu}} di \right)^{\frac{\theta_\mu}{\theta_\mu - 1}}.$$

The solution is

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta_\mu} Y_t,$$

and the zero profit condition for the final goods producer implies

$$P_t = \left(\int_0^1 P_t(i)^{1 - \theta_\mu} di \right)^{\frac{1}{1 - \theta_\mu}}.$$

A.3 Intermediate goods producers

The intermediate goods firms own capital, and hire labor in the labor market. Capital accumulation and labor hiring are subject to a joint convex adjustment cost function $\Phi(I_t(i), K_t(i), H_t(i), N_{t-1}(i))$:

$$\Phi = \frac{\phi_1}{2} \left(\frac{I_t(i)}{K_t(i)} - \delta_{K,0} \right)^2 K_t(i) + \frac{\phi_2}{2} \left(\frac{H_t(i)}{N_{t-1}(i)} - \delta_N \right)^2 N_{t-1}(i)$$

where $I_t(i)$ denotes investment of firm i , $H_t(i)$ denotes hiring. The production has to satisfy demand:

$$\left(\frac{P_t(i)}{P_t} \right)^{-\theta_\mu} Y_t \leq (U_t(i) K_t(i))^\alpha (Z_t N_t(i))^{1-\alpha} - \Psi$$

where Z_t is the TFP, and U_t is capital utilization rate chosen by the firm. Ψ is a fixed cost that ensures zero profit in the non-stochastic steady state. Assume that the firm is an all-equity firm, so the dividend payment is

$$\begin{aligned} D_t^E(i) &= \left(\frac{P_t(i)}{P_t} \right)^{1-\theta_\mu} Y_t - W_t N_t(i) - I_t(i) \\ &\quad - \Phi(I_t(i), K_t(i), H_t(i), N_{t-1}(i)) - \frac{\phi_P}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} \frac{1}{\Pi} - 1 \right)^2 Y_t. \end{aligned} \quad (\text{A.8})$$

Taking P_t and Y_t as given, the intermediate goods firm i optimizes the expected discounted dividends flow by choosing $\{I_t(i), H_t(i), U_t(i), P_t(i)\}_{t \geq 0}$

$$J_t = \max \mathbf{E}_t \left[\sum_{s=1}^{\infty} M_{t+s} D_{t+s}^E(i) \right] \quad (\text{A.9})$$

subject to the capital and labor accumulation equations

$$K_{t+1}(i) = I_t(i) + (1 - \delta_K(U_t(i))) K_t(i), \quad (\text{A.10})$$

$$N_t = H_t + (1 - \delta_N) N_{t-1}. \quad (\text{A.11})$$

Capital depreciation rate $\delta_K(U_t(i))$ depends on the current capital utilization rate. The functional form of $\delta_K(U_t(i))$ is

$$\delta_K(U_t(i)) = \delta_{K,0} + \delta_{K,1}(U_t(i) - U) + \frac{\delta_{K,2}}{2}(U_t(i) - U)^2.$$

Let the Lagrange multipliers associated with Equations A.8, A.10, and A.11 be Ξ_t , λ_t^K , and λ_t^N . The first order conditions for the intermediate goods firm i are:

$$-\Phi_H + \lambda_t^N = 0 \quad (\text{A.12})$$

$$\alpha \Xi_t (U_t K_t)^{\alpha-1} (Z_t N_t)^{1-\alpha} = [\delta_{K,1} + \delta_{K,2}(U_t - U)] \lambda_t^K \quad (\text{A.13})$$

$$\lambda_t^K = \mathbf{E}_t \left[M_{t+1} \left(\alpha \Xi_{t+1} U_{t+1}^\alpha K_{t+1}^{\alpha-1} (Z_{t+1} N_{t+1})^{1-\alpha} - \Phi_K(t+1) + (1 - \delta_K(U_{t+1})) \lambda_{t+1}^K \right) \right] \quad (\text{A.14})$$

$$\lambda_t^N = (1 - \alpha) \Xi_t (U_t K_t)^\alpha (Z_t N_t)^{-\alpha} - W_t + \mathbf{E}_t \left[M_{t+1} \left(-\Phi_N(t+1) + (1 - \delta_N) \lambda_{t+1}^N \right) \right] \quad (\text{A.15})$$

$$\begin{aligned} \phi_P \left(\frac{P_t(i)}{P_{t-1}(i)} \frac{1}{\Pi} - 1 \right) \left(\frac{P_t}{P_{t-1}(i)} \frac{1}{\Pi} \right) &= (1 - \theta_\mu) \left(\frac{P_t(i)}{P_t} \right)^{-\theta_\mu} + \theta_\mu \Xi_t \left(\frac{P_t(i)}{P_t} \right)^{-\theta_\mu-1} \\ &+ \phi_P \mathbf{E}_t \left[M_{t+1} \frac{Y_{t+1}}{Y_t} \left(\frac{P_{t+1}(i)}{P_t(i)} \frac{1}{\Pi} - 1 \right) \left(\frac{P_{t+1}(i)}{P_t(i)} \frac{1}{\Pi} \frac{P_t}{P_t(i)} \right) \right] \end{aligned} \quad (\text{A.16})$$

A.4 Monetary authority

The economy is cashless and the monetary authority sets the nominal interest rate according to the rule

$$r_t^n = r^n + \rho_\pi (\pi_t - \pi) + \rho_y \Delta y_t, \quad (\text{A.17})$$

where $r_t^n = \ln(R_t^n)$ denotes the net nominal interest rate, $\pi_t = \ln(\Pi_t) = \ln(P_t/P_{t-1})$ denotes the net inflation rate, and $\Delta y_t = \ln(Y_t/Y_{t-1})$ denotes the net growth rate of real output. The nominal interest rate affects expected inflation and consumption decisions through the Euler equation

$$1 = R_t^n \mathbf{E}_t \left[M_{t+1} \frac{1}{\Pi_{t+1}} \right].$$

A.5 Equilibrium

In equilibrium, ex ante homogeneity of the intermediate goods firms and the Rotemberg price adjustment cost imply that all intermediate goods firms make the same decisions: $P_t(i) = P_t$, $H_t(i) = H_t$, $I_t(i) = I_t$, $K_t(i) = K_t$, $N_t(i) = N_t$, $U_t(i) = U_t$. The equilibrium is defined in the standard way: subject to the relative prices, household maximizes utility, firms maximize profits, and the relative prices clear the markets. The equations characterizing the equilibrium are A.1, A.2, A.3, A.4, A.5, A.6, A.7, A.8, A.10, A.11, A.12, A.13, A.14, A.15, A.16, A.17.

B Data

The time series of macroeconomic variables are obtained from FRED. I obtain the following data series: population (B230RC0Q173SBEA, thousands), VXO (VXOCLS), real GDP per capita (A939RX0Q048SBEA, chained 2012 dollars), real personal consumption expenditures per capita: goods: nondurable goods (A796RX0Q048SBEA, chained 2012 dollars), real personal consumption expenditures per capita: services (A797RX0Q048SBEA, chained 2012 dollars), real personal consumption expenditures per capita: goods: durable goods (A795RX0Q048SBEA, chained 2012 dollars), nonfarm business sector: average weekly hours (PRS85006023, index 2012 = 100), GDP implicit price deflator (GDPDEF, index 2012 = 100), private nonresidential fixed investment (PNFI, billions of dollars), 3-month treasury bill: secondary market rate (TB3MS), national income: compensation of employees, paid (COE, billions of dollars).

I transform the raw data as following.

- Nominal wage = national income: compensation of employees, paid / (population*nonfarm business sector: average weekly hours).

Real wage = nominal wage*100 / GDP deflator.

- Real private nonresidential fixed investment per capita = private nonresidential fixed investment * 1,000,000 / population / GDP deflator * 100.

Real investment = real private nonresidential fixed investment per capita + real personal consumption expenditures per capita: durable goods.

- Real consumption = real personal consumption expenditures per capita: goods: nondurable goods + real personal consumption expenditures per capita: services

- Inflation rate = $[\ln(\text{GDP deflator}) - \ln(\text{GDP deflator}_{t-1})] * 4$.
- Real interest rate = 3-month treasury bill rate - inflation rate.
- Except for the real interest rate, all other variables enter the VAR in log levels.

C Additional results

Impulse responses of the baseline model Figure C.1 plots the impulse responses of all macroeconomic aggregates to an increase in uncertainty in the flexible price baseline model. All variables decrease on impact, peaking 4 quarters after the shock. Notably, the stochastic discount factor decreases by more than 10% relative to the steady state. This means that the market discounts risky cashflows by a much larger discount rate.

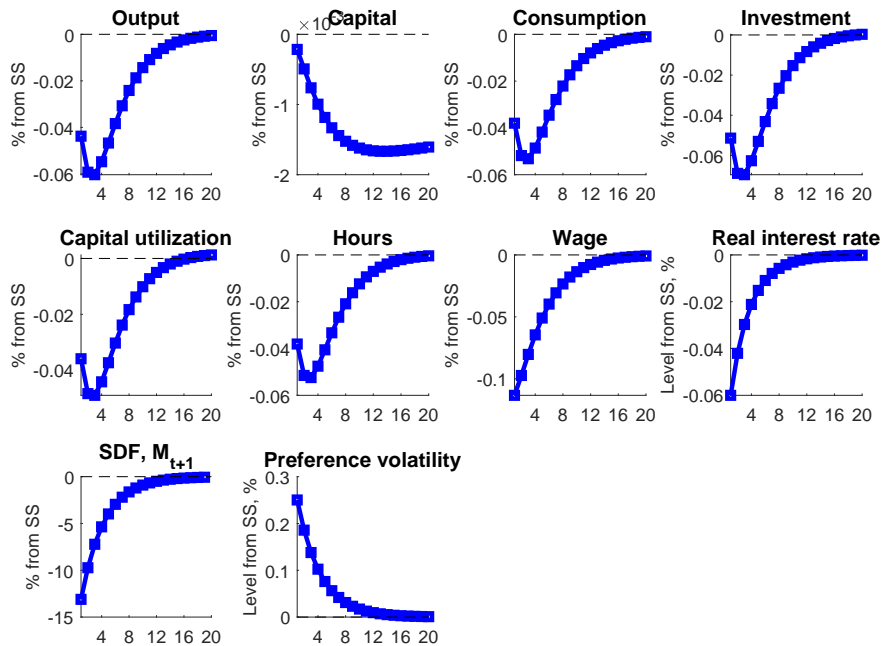


Figure C.1: Model implied impulse responses to a positive uncertainty shock, baseline model with flexible prices.

Figure C.2 plots the impulse responses of all macroeconomic aggregates in the sticky price baseline model. Compared with the flexible price model, the real variables have roughly the same magnitude of responses except that real investment shows larger peak response. Impulse response

of hours worked is hardly affected by adding price stickiness. Markup increases and inflation decreases due to the decline in real wage and consumption.

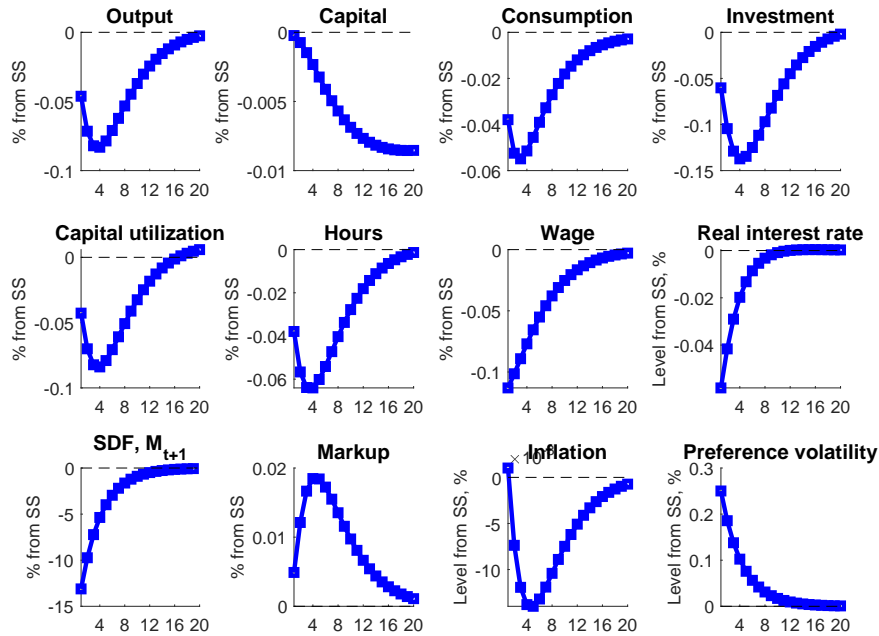


Figure C.2: Model implied impulse responses to a positive uncertainty shock, baseline model with sticky prices.

Impulse responses of the model without labor adjustment cost Figure C.3 plots the impulse responses of all macroeconomic aggregates in the flexible price model without labor adjustment cost. The model has a key feature shared by many macroeconomic models with an RBC core: the labor demand is static. On impact of the increased uncertainty, consumption drops due to the precautionary motive. Hours worked increase because labor supply shifts out while labor demand stays unchanged. Output also increases because equilibrium hours increase while capital and productivity are unchanged. Investment also increases to because the real interest rate decreases. The decline in real interest rate is three times as much as in the baseline model, while the decline in the stochastic discount factor is of the same magnitude.

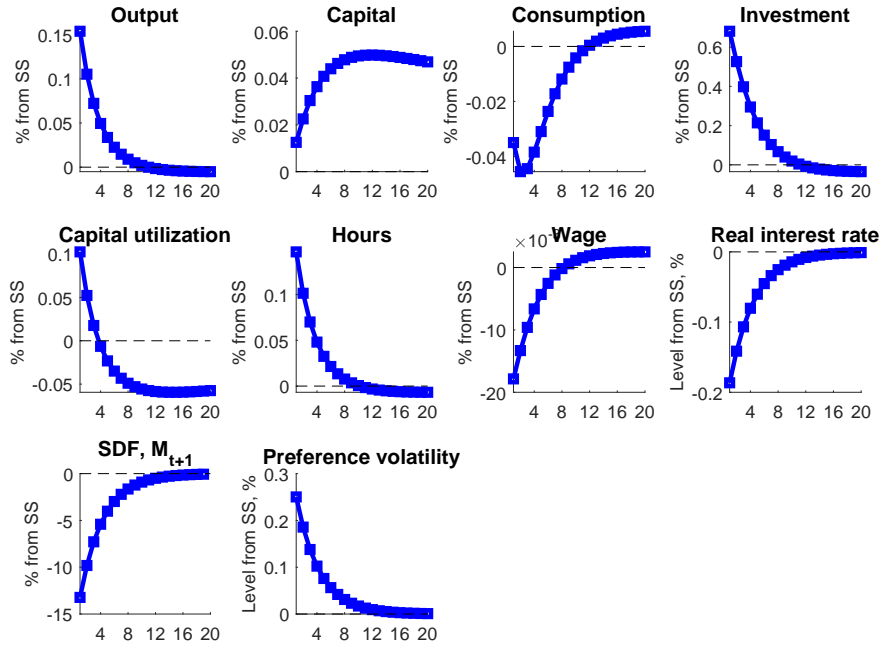


Figure C.3: Model implied impulse responses to a positive uncertainty shock, no-labor-adjustment-cost model with flexible prices.

Figure C.4 plots the impulse responses of all macroeconomic aggregates in the sticky price model without labor adjustment cost. Real consumption declines by roughly the same amount as in the flexible price model. Under sticky prices, the markup endogenously increases, depressing labor demand. Equilibrium hours drop. Real interest rate drops by 6 basis points, which is similar as in the baseline model and way less in magnitude than in the RBC model without labor adjustment cost.

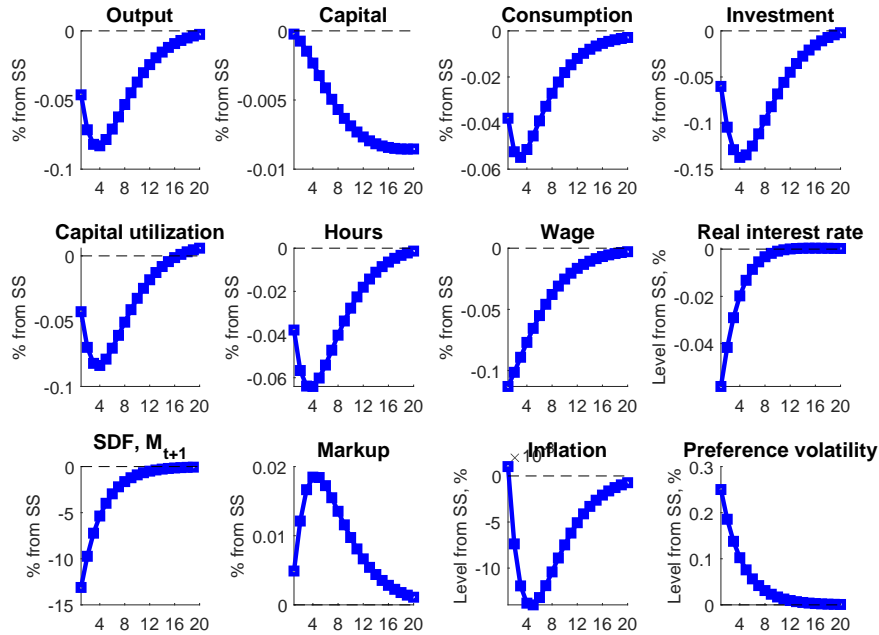


Figure C.4: Model implied impulse responses to a positive uncertainty shock, no-labor-adjustment-cost model with sticky prices.

Impulse responses of the model with labor adjustment cost and risk neutral firms Figure C.5 plots the impulse responses of all macroeconomic aggregates in the flexible price model with risk neutral firms. The purpose of this exercise is to isolate the effect of the stochastic discount factor on labor demand. The declines in consumption and the stochastic discount factor are the same as in the baseline model, suggesting that the consumption Euler equation is hardly affected by the firms being risk neutral. Wage falls by 0.1%, which is the same as in the baseline model. By the intratemporal Euler equation 3.4, labor supply curve should be at the same position as in the baseline model. On the firm side, the absence of the stochastic discount factor makes the expected future value of the marginal labor less vulnerable to the uncertainty shock. Investment and labor demand both increase. The increased dispersion of future dividend streams does not decrease the firm's expectation of the value of future labor.

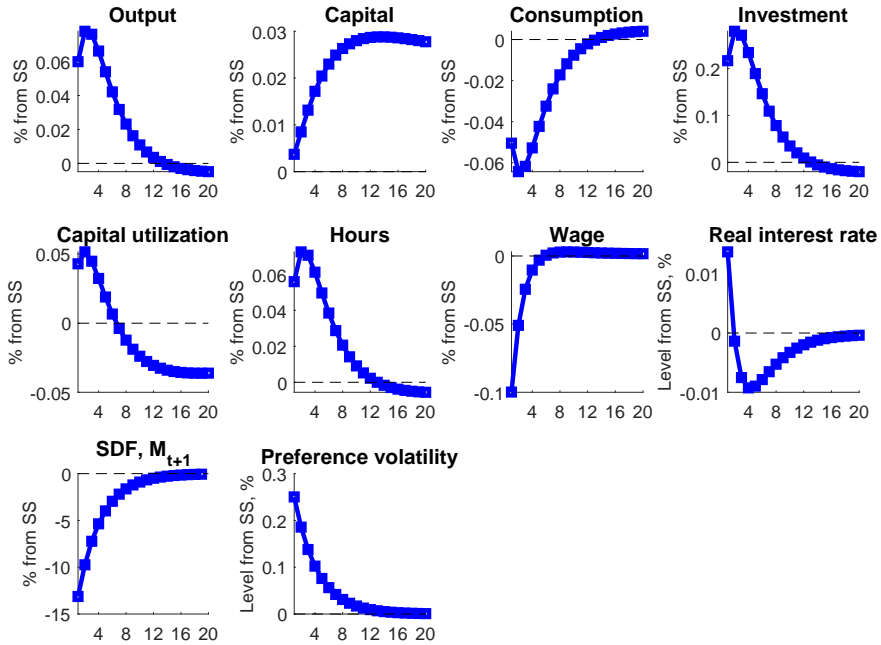


Figure C.5: Model implied impulse responses to a positive uncertainty shock, risk-neutral-firm model with flexible prices.

Figure C.6 plots the impulse responses of all macroeconomic aggregates in the sticky price model with risk neutral firms. Adding price stickiness does not correct the comovements. Markup increases as in other models, but inflation also increases. Consumption declines on impact. Being demand-driven, output falls mainly due to the decline in capital utilization. Labor demand does not offset the increase in labor supply, so equilibrium hours increase. Again, the absence of the stochastic discount factor insulates the firm's expected value of future labor from the uncertainty shock. Price stickiness does not resolve this issue.

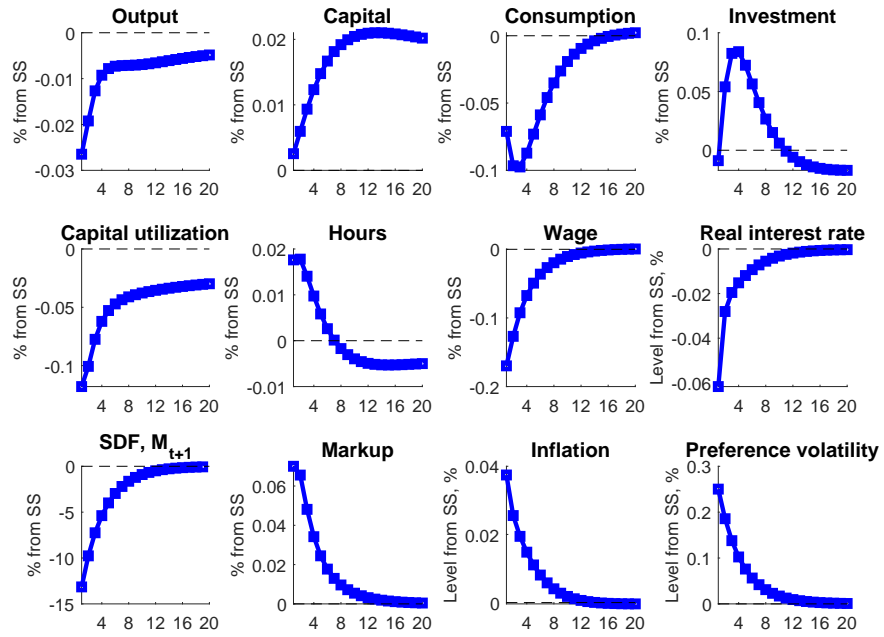


Figure C.6: Model implied impulse responses to a positive uncertainty shock, risk-neutral-firm model with sticky prices.